Noise thermometry and electron thermometry of a sample-on-cantilever system below 1 Kelvin

A. C. Bleszynski-Jayich and W. E. Shanks
Department of Physics, Yale University, New Haven, Connecticut 06520, USA

J. G. E. Harris
Departments of Physics and Applied Physics, Yale University, New Haven, Connecticut 06520, USA

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We have used two types of thermometry to study thermal fluctuations in a microcantilever-based system below 1 K. We measured the temperature of a cantilever’s macroscopic degree of freedom (via the Brownian motion of its lowest flexural mode) and its microscopic degrees of freedom (via the electron temperature of a metal sample mounted on the cantilever). We also measured both temperatures’ response to a localized heat source. We find that it is possible to maintain thermal equilibrium between the two temperatures and a refrigerator down to at least 300 mK. These results are promising for ongoing experiments to probe quantum effects using micromechanical devices. © 2008 American Institute of Physics. [DOI: 10.1063/1.2821828]

There are at least two distinct “temperatures” relevant to the performance of mechanical devices. The first is the effective temperature associated with the device’s Brownian motion. This thermomechanical noise temperature $T_n$ sets a fundamental limit to the device’s force sensitivity. It is relevant in magnetic resonance force microscopy (MRFM),1,2 atomic force microscopy, and torque magnetometry.3–5 It also sets limits on the observation of quantum effects in mechanical oscillators.6–9 As a result there is a considerable interest in lowering this “Brownian” temperature via cryogenics10–12 and/or cold damping techniques such as laser cooling.13–18

The second important temperature is that of the cantilever’s microscopic degrees of freedom. For sample-on-cantilever experiments, this sets the temperature of the sample attached to the cantilever, $T_e$, and is important for MRFM and torque magnetometry experiments.3–5,19,20

In principle, both $T_n$ and $T_e$ can be lowered by placing the cantilever in contact with a thermal bath (i.e., a refrigerator) at temperature $T_B$. However, thermal equilibrium between the bath, the lever’s Brownian motion, and a sample affixed to the lever is not assured. Factors preventing equilibration include the extreme aspect ratio of typical cantilevers, their high quality factors, the insulating nature of most cantilever materials, and the injection of heat by the lever’s readout mechanism (e.g., a laser).

Previous experiments have studied $T_e$ of a sample at the end of a gold-coated cantilever between 4 and 16 K.19 In other experiments, $T_n$ has been cooled in a refrigerator to 200 mK in micromechanical systems10 and to 56 mK in nanomechanical devices.12 We are not aware of any direct measurements of both $T_n$ and $T_e$ in a single system.

Here, we present measurements of $T_n$ of a cantilever and $T_e$ of an aluminum grain attached at the end of the cantilever. $T_n$ is measured via the cantilever’s Brownian motion, while $T_e$ is measured via the grain’s superconducting critical field $H_c$. We also measure the response of $T_n$ and $T_e$ to the laser interferometer which monitors the cantilever. We find that $T_n$ and $T_e$ remain in good contact with each other and with $T_B$ for temperatures down to 300 mK and laser powers $P_{\text{inc}}$ below $\sim 150$ nW. At higher laser powers, $T_e$ and $T_n$ increase above $T_B$ in a manner consistent with diffusive phonon-mediated heat transport through the cantilever.

These experiments were performed in a $^3$He refrigerator.21 A schematic of the setup is shown in Fig. 1(a). A single crystal silicon cantilever22 of length $L=500$ μm, width $w=100$ μm, thickness $t=1$ μm, and doping of $\sim 10^{18}$ cm$^{-3}$ is mounted on a piezoelectric actuator and thermally linked to the refrigerator. A fiber optic interferometer is used to measure the cantilever deflection $x$. This interferometer is formed between the cantilever and the cleaved face of a single mode optical fiber $\sim 100$ μm from the cantilever. The interferometer uses a laser wavelength $\lambda=1550$ nm.

The noise temperature is determined by measuring the mean square displacement $\langle x^2 \rangle$ of the cantilever’s free end. From the equipartition theorem $T_n=k_0\langle x^2 \rangle/k_B$, where $k_B$ is the cantilever’s spring constant. To obtain an absolute measurement of the displacement $x$, we calibrate the interferometer signal by applying a sinusoidal drive to the piezoelectric actuator and measuring the fundamental Fourier component of the interferometer signal on a lock-in amplifier as a function of the drive amplitude. The data are shown in Fig. 2(a).

To fit these data, we note that the optical field at the photodiode has two sources: light reflected from the fiber’s end $E_1$ (which we assume is constant) and light reflected from the cantilever $E_2$ (we take $E_1$ and $E_2$ to be complex). As the cantilever deflects, the phase of $E_2$ changes, producing

FIG. 1. (Color online) (a) Experimental schematic. Laser interferometry is used to monitor the deflection of a cantilever (shown here with an Al particle attached). (b) SEM image showing an Al grain attached at the end of a Si cantilever. Scale bar is 100 μm.
the interferometer signal. The cantilever deflection also modulates the amplitude of $E_2$ since the amount of reflected light coupled back into the fiber varies with the cantilever’s angle relative to the fiber axis. This effect is small enough to be expanded to first order in $x$. Thus, we can write the total field at the photodiode as $E_{\text{tot}} = E_1 + E_2^{(0)}(1 + e(x(t) - x_0))e^{2ikx_0}t$, where $k = 2\pi/\lambda$, $e = \partial E_2/\partial x|_{x=x_0}$, $e(x(t)-x_0) \ll 1$, $x_0$ is the equilibrium position of the cantilever, and $E_2^{(0)}$ is the value of $E_2$ for $x = x_0$. The time dependent cantilever position is $x(t) = x_0 + x_1 \sin(2\pi ft)$, where $x_1$ is the amplitude of the cantilever’s oscillation and $f$ is its frequency. The lock-in signal $V_{\text{lock-in}}$ is proportional to the Fourier component of $|E_{\text{tot}}|^2$ at $f$:

$$V_{\text{lock-in}} \propto 2E_2^2 x_1 - 4E_1E_2 \sin(2kx_0)J_1(2kx_1)$$

$$+ 2E_1E_2 \cos(2kx_0)x_1\{J_0(2kx_1) - J_2(2kx_1)\},$$

where $J_n$ is the $n^{th}$ Bessel function of the first kind, and we have kept terms linear in $e$. Fitting the data in Fig. 2(a) to this expression allows us to convert $V_{\text{lock-in}}$ to an absolute displacement $x$ in terms of the known laser wavelength.

We then monitor the interferometer signal when no drive is applied to the piezoactuator and use the calibration described above to convert this signal to $S_x$, the power spectral density of the cantilever’s undriven motion [Fig. 2(b)]. The data are fit to the response function of a damped harmonic oscillator, giving a quality factor $Q = 70,000$ and a resonant frequency $f_0 = 7276$ Hz. The baseline in Fig. 2(b) is a factor of 4 above the photon shot noise. The cantilevers used in the $T_n$ measurements did not have an Al grain attached.

The area under the fit in Fig. 2(b) (after subtracting the baseline) is $\langle \chi^2 \rangle$. We measured $\langle \chi^2 \rangle$ at refrigerator temperatures between $T_b = 300$ mK and $4.2$ K [Fig. 2(c)]. The linear dependence of $\langle \chi^2 \rangle$ on $T_b$ and its extrapolation to zero at $T_b = 0$ K confirm that the force noise driving the cantilever is thermal and, hence, that the motion in Fig. 2(b) is Brownian. Importantly, Fig. 2(c) indicates that $T_n$ remains in equilibrium with the refrigerator down to $300$ mK for $P_{\text{inc}} = 150$ nW. The slope of the fit yields $k_b = 0.02$ N/m, in agreement with the manufacturer’s specifications.

To determine the electron temperature of a sample on a cantilever, we measure a nominally identical cantilever on the same chip to which we attached a $\sim 10$ $\mu$m diameter Al grain (99.99% pure). A scanning electron micrograph (SEM) of a cantilever with attached Al grain is shown in Fig. 1(b). We drive the cantilever in a phase-locked loop and measure $f_0$ as a function of applied magnetic field $H$, as shown in Fig. 3(a). The red data (positive sweep of $H$) and blue data (negative sweep) have been shifted slightly to correct for hysteresis in the magnet.

Figure 3(a) shows that below a critical field $H_0$ (indicated on the graph), $f_0 \propto H^2$, while above $H_c$, $f_0$ abruptly drops back to its $H=0$ value and ceases to depend on $H$. We interpret this jump as the grain’s transition from the superconducting state to the normal state. The quadratic dependence of $f_0$ on $H$ in the superconducting state arises from a combination of the grain's Meissner effect, which induces a magnetic moment $m \propto H$, and the grain’s nonspherical shape. This combination causes the grain’s energy $E$ to depend on the angle $\theta$ its principal axis makes with $H$. The dependence of $E$ results from the energy associated with the grain’s demagnetizing fields $E_D = \frac{1}{2}\mu_0 m^2 N(\theta)$, where $N(\theta)$ is a shape anisotropy factor. The shift in $f_0$ is proportional to $\partial^2 E / \partial \theta^2$ which, in turn, is proportional to $H^2$. The hysteresis seen in Fig. 3(a) is due to supercooling of the Al particle. Figure 3(b) shows $H_c$ as a function of $T_n$. Fitting the data using Bardeen-Cooper-Schrieffer (BCS) theory (which predicts $H_c(T) = H_c(0)[1-(T/T_c)^2]$) yields $H_c(0) = 123$ G and $T_c(0) = 1.19$ K. This value of $T_c$ agrees with the value for
and 28 meters for rect measurement of absorption. The laser spot, where \( \frac{H_n}{H_{9251}} \) meters to measure the response of cryogenic temperatures. We note that measurements of optical loss in similarly doped Si at rect measurements of optical loss in similarly doped Si at 300 mK. Given the signal-to-noise ratio in Fig. (b), this approach could be used with much smaller samples, including microfabricated devices. We also determined the optical absorption of the cantilever.

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