Optomechanical systems couple mechanical degrees of freedom to radiation fields and constitute a rapidly evolving field of current research (reviewed in [1]). These systems provide new means to manipulate both the light field and the mechanical motion. Apart from the hope of eventually exploring the quantum regime of mechanical motion, there have been several studies of the complex nonlinear dynamics of these systems [2–6]. An exciting recent development is the introduction of setups with multiple coupled optical (and vibrational) modes, pointing the way toward integrated optomechanical circuits [7–10]. In this article, we show how the application of an external mechanical drive to these structures can open up the whole domain of strongly driven two-level and multilevel systems to the field of optomechanics. As a concrete example of such a mechanically driven coherent photon dynamics, we consider the system recently realized in [7,11], where we show that a vibrating membrane inside an optical cavity can shuttle photons between two optical modes. We predict Autler-Townes splittings due to Rabi processes and Landau-Zener-Stuckelberg dynamics originally known from atomic two-state systems. Observing an Autler-Townes splitting indicative of Rabi dynamics. For large drive, we show that this system can be shuttled between the two halves of the cavity. For modest driving strength we predict the possibility of observing an Autler-Townes splitting indicative of Rabi dynamics. For large drive, we show that this system displays Landau-Zener-Stuckelberg dynamics originally known from atomic two-state systems.

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FIG. 1. (Color online) (a) Setup. A dielectric membrane couples two modes \(a_L\), \(a_R\) inside a cavity. The left-hand side is excited by a laser \(\omega_L\) while the transmission to the right is recorded. (b) Optical resonance frequency as function of displacement. The membrane’s displacement linearly changes the bare mode frequencies (dashed). Due to the coupling \(g\), there is an avoided crossing of the eigenfrequencies \(\omega_{\pm}\) (black). The membrane is driven, with \(x(t) = A \cos(\Omega t) + x_0\) (blue; thicker).

\[
\hat{H}_{\text{sys}} = \hbar \omega_0 \left[ 1 - \frac{x(t)}{T} \right] \hat{a}_L \hat{a}_L^\dagger + \hbar \omega_0 \left[ 1 + \frac{x(t)}{T} \right] \hat{a}_R \hat{a}_R^\dagger
+ \hbar g (\hat{a}_L^\dagger \hat{a}_R + \hat{a}_R^\dagger \hat{a}_L) + \hat{H}_{\text{drive}} + \hat{H}_{\text{decay}}.
\]
membranes range between 100 kHz and 1 MHz. We point out that here Ω need not coincide with the membrane’s eigenfrequency but depends only on the driving. In the experimental scheme, we propose to optically drive the left-hand side of the cavity with a laser of tunable frequency ωL and measure the transmission T to the right with a photo detector placed on the other side of the cavity [see Fig. 1(a)]. The mechanical driving Eq. (2) might be realized by mounting the membrane on a piezo actuator.

Using input-output theory, the equations of motion for the average fields aL = ⟨aL⟩ and aR = ⟨aR⟩ read

$$\frac{d}{dt} a_L = \frac{-\bar{x}(t) a_L + g a_R - \kappa}{i} a_L - \sqrt{\kappa} b_L^\dagger(t)$$

$$\frac{d}{dt} a_R = \frac{+\bar{x}(t) a_R + g a_L - \kappa}{i} a_R,$$

with the cavity decay rate κ for each of the modes, and laser drive $b_L^\dagger(t) = e^{-i\Delta_1 L t} b_L^\dagger$ with amplitude $b_L^\dagger$. Here, we used a rotating frame, with laser detuning from resonance $\Delta_L = \omega_L - \omega_0$. The displacement is written in terms of a frequency via $\bar{x}(t) = (\omega_0/1) x(t)$; likewise for $A, \bar{x}_0$. The transmission to the right, $T(t) = \kappa (\langle a^\dagger_R(t) a_R(t) \rangle / \langle |b_L^\dagger|^2 \rangle)^2$, can be expressed as

$$T(t) = \kappa^2 \left| \int_{-\infty}^t G(t, t') e^{-i\Delta_1 L t' - (\kappa/2)(t-t')} dt' \right|^2,$$

where the phase factor includes laser drive and cavity decay, while the Green’s function $G(t, t')$ describes the amplitude for a photon to enter the cavity from the left at time $t'$ and to be found in the right cavity mode later at time $t$. Technically, $G(t, t')$ is found by setting $\kappa = 0$ in Eq. (3) and solving for $a_R(t')$ with the initial conditions $a_L(t') = 1, a_R(t') = 0$.

Figure 2(a) displays the time-averaged transmission depending on $\bar{x}_0$ and $\Delta_0$ without mechanical drive ($A = 0$). The spectrum displays the two hyperbola branches $\omega_{\pm}$ [Fig. 1(b)]. Transmission is largest at the avoided crossing where photons can most easily tunnel from the left into the right mode. For modest drive $A < \Omega$, Fig. 2(b) shows an Autler-Townes splitting [22,25] of the two hyperbola branches $\omega_{\pm}$. Indeed, if resonant, the mechanical drive induces Rabi oscillations between the two photon branches, at a Rabi frequency $\Omega_1 \simeq g \tilde{A} / \Omega$ [see Eq. (4)], leading to a corresponding splitting in the spectroscopic picture. Furthermore, the mechanically assisted process enables high transmission even farther away from the anti-crossing. For larger drive amplitudes [see Fig. 2(c)], the dynamics becomes more involved as mechanical sidebands arise and interact with each other. In the following, we will focus on the dynamics of the strong driving regime.

Figure 3 shows numerical results for $A \gg \Omega, g$ and experimentally accessible parameters (for $g/2\pi = 1$ MHz, $l = 1$ cm, $\omega_0/2\pi = 3 \times 10^{14}$ Hz; $A = 60$ g corresponds to an oscillation amplitude $A = 2$ nm). We first give an intuitive description of why finite transmission $T$ can be observed only if $\bar{x}_0$ is a multipole of $\Omega$, and we comment later on the modulation as a function of $A$. $T$ is determined by two subsequent processes: first, the laser has to excite the left mode; second, the internal dynamics must be able to transfer photons into the right one. In general, both processes are inelastic and therefore require energy to be exchanged between the light field and the mechanics. The left mode’s frequency is oscillating around the time-averaged value $-\bar{x}_0$. Hence, the resonance condition to excite the left mode reads

$$\Delta_L + m \Omega = -\bar{x}_0,$$

[see Fig. 4(a)]. Here, $m \Omega$ is an adequate multiphonon transition. The width of the individual resonances is determined by $\kappa$. The subsequent process displays the physics of LZS dynamics: in general, if a parameter in a two-state system is swept sufficiently fast through an avoided crossing, the system may undergo an LZ transition into the other eigenstate [12,13]. Here, for periodic sweep, we face iterated transitions. The first LZ transition splits the photon state into a coherent superposition; the two amplitudes gather different phases and interfere the next time the system transverses the avoided crossing. For two-state systems the resulting interference patterns in the state population are known as Stueckelberg oscillations [14]. The condition for constructive interference can also be phrased in terms of an additional multiphonon transition that transfers a photon from the left mode with average frequency $-\bar{x}_0$ to the right one at $+\bar{x}_0$.

$$n \Omega = 2\bar{x}_0.$$

We find transmission only if both conditions are met. We note that the coupling $g$ between modes does not enter here. We will come back to this point later.

To derive these resonance conditions as well as to understand the dependence on $A$, in the following, we calculate an approximate, analytic expression for the transmission.

FIG. 2. (Color online) Density plot for the time-averaged transmission depending on mean position $\bar{x}_0$ and laser detuning $\Delta_L$ for cavity decay rate $\kappa = 0.1 \ g$. (a) Without mechanical drive $A = 0$, the spectrum is given by $\omega_{\pm}$ [see Fig. 1(b)]. (b) Autler-Townes splitting of the cavity frequency $\omega_{\pm}$ due to mechanical motion, $A = 0.2 \ \Omega$. For every position $\bar{x}_0$ the mechanical drive frequency is set to $\Omega = 2 \sqrt{\bar{x}_0^2 + \tilde{x}_0^2}$ such that it is always resonant with the difference between the two optical mode frequencies $\omega_{\pm}$. The splitting is set by the Rabi frequency $\Omega_1 \simeq g \tilde{A} / \Omega$ [see Eq. (4)]. (c) Plot as in panel (b) but for stronger drive $A = 1.6 \ \Omega$. Mechanical sidebands, displaced by $\pm \Omega$, become visible and interact.
From Eq. (3), the Green’s function \( G(t, t’) \), required for the transmission \( (4) \), is found to be
\[
G(t, t’) = e^{-i\phi(t’)} \sum_n J_n(\hat{A}(\Omega)) e^{i\Delta_n t’}.
\]
\[
(7)
\]
where we have split off a phase \( \phi(t’) = (\hat{A}(\Omega)) \sin(\Delta_0 t’) \), and \( \tilde{a}_L(t, t’) \) is a solution to
\[
\frac{d}{dt} \tilde{a}_L = g \sum_n J_n(\hat{A}(\Omega)) e^{i\Delta_n t’}.
\]
\[
(8)
\]
with \( t \geq t’ \) and initial condition \( \tilde{a}_L(t’, t’) = 0 \), \( \tilde{a}_L(t’, t’) = 1 \). We now show that the two multiphonon processes introduced above correspond to the two factors in Eq. (7). The term \( e^{-i\phi(t’)} \) describes the initial excitation, where the amplitude for a transfer of \( m \) phonons is set by the Bessel function \( J_m(\hat{A}(\Omega)) \). Second, the internal dynamics described by \( \tilde{a}_L(t, t’) \) is expressed in terms of a two-level system with time-dependent coupling \( ge^{i2\phi(t’)} = g \sum_n J_n(\hat{A}(\Omega)) e^{i\Delta_n t’} \). Thus, the strength of the second multiphonon transition \( n \Omega \) in Fig. 4(a) is determined by a Bessel function \( J_n(2\hat{A}(\Omega)) \). As a special case, this also describes the Autler-Townes splitting at small drive. This can be calculated from (8) using an interaction picture representation and considering the time-dependent coupling only up to \( J_1 \), yielding an effective transition frequency \( 2\sqrt{g_0^2 + \tilde{x}_0^2} \), with \( g_0 = g J_0(2\hat{A}(\Omega)) \), and a Rabi frequency \( g_1 = g J_1(2\hat{A}(\Omega)) \).

In the case of LZS dynamics (i.e., strong drive), for sufficiently large amplitudes only one of the harmonics of \( g \sum_n J_n(2\hat{A}(\Omega)) e^{i\Delta_n t’} \) will be in resonance with the system. This corresponds to leading-order perturbation theory within the Floquet approach [26] applied to Eq. (8). In this case, Eq. (8) simplifies to the problem of a two-state system with harmonic drive at \( n \Omega \) and effective coupling constant
\[
g_n = g J_n(2\hat{A}(\Omega)).
\]
\[
(9)
\]
To estimate when this approximation becomes appropriate, we note that for a driven undamped two-state system the width of the power-broadened resonance is set by the Rabi frequency. Thus, Eq. (8) yields a series of resonance peaks at \( \tilde{x}_0 = n\Omega/2 \), and they become separated if \( 4g_n < \Omega \). Using the asymptotic form for large arguments \( \tilde{\Delta}/\tilde{\Omega} \gg 1 \), \( J_n(y) \approx \sqrt{\frac{\pi}{2x}} \cos(y - \frac{\pi}{4} - \frac{\Delta}{\Omega}) \), we find the resonance approximation to hold whenever \( g^2 < (\pi/16)\Omega^2 \). This is clearly fulfilled for the parameters in Fig. 3. The amplitude for a transfer of \( n \) phonons \( g_n \) contains the Rabi frequency \( g \), which is much smaller than the bare splitting \( g \) for \( \tilde{\Delta} \gg \Omega \). This explains why the resonance conditions (5) and (6) involve the bare optical mode frequencies \( \pm\tilde{x}_0 \) instead of the adiabatic eigenfrequencies \( \omega_m \).

We insert (10) into (4), taking into account the sum over independent contributions with \( n \) quanta. In the resolved sideband regime \( \Omega > \kappa \), the integration of (4) selects a specific \( m \) for the excitation process [see Eq. (5)]. We find an approximate expression (displayed here for the special case \( \Delta_L = 0 \), where \( n = 2m \),
\[
T = \left( \frac{\kappa}{g} \right)^2 \sum_m \left[ J_m(\frac{\hat{A}}{\Omega}) \right]^2
\]
\[
\times \left[ J_{2m}(\frac{\hat{A}}{\Omega}) \right]^2 + \left[ J_{m}(\frac{2\hat{A}}{\Omega}) \right]^2 \right]^2,
\]
\[
(11)
\]
that fully captures the numerical results shown in Fig. 3. In contrast to \( J_m(\hat{A}(\Omega)) \), the LZY dynamics, characterized by \( J_n(2\hat{A}(\Omega)) \), involves \( 2\hat{A} \) as it is determined by the phase difference gathered between LZ transitions. If we were to increase \( \Delta_L \) in Fig. 3, we would tune out of resonance and the transmission would vanish everywhere. For \( \Delta_L = \Omega/2 \), (5) and (6) can be met for \( \tilde{x}_0 \) being an odd multiple of \( \Omega/2 \). For \( \Delta_L = \Omega \), we recover the resonances of Fig. 3; however, the entire plot would be shifted in \( \hat{A} \) by \( \pi\Omega/2 \) due to the changed index of the Bessel function \( J_m \). The qualitative result at \( \Delta_L = 2\hat{A} \) finally resembles the one shown in Fig. 3.
FIG. 4. (Color online) (a) Multiphonon transition picture. To see transmission, two processes are involved: First (magenta, labeled $m\Omega$), excitation of the left cavity mode by the laser drive at $\Delta_L$, supported by $m$ phonons. Second (red, labeled $n\Omega$), a suitable $n$-quantum multiphonon transition to transfer a photon from the left into the right mode. (b) Density plot for time-averaged transmission as a function of $\bar{x}_0$ and $A \simeq g$. Further parameters as in Fig. 3.

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